Mathematics II: Probability Theory

Naples Ph.D. in Economics: Fall 2025

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1 Overview

Description. This course provides a graduate-level introduction to Probability Theory and its foundations in Measure Theory. Throughout the course, we will explore several connections to Mathematical Analysis. Although we will not compromise on rigor, we will cover the course's content in a way that the concepts and techniques you will learn are broadly applicable to a variety of ideas and problems in Economic Theory, Econometrics, and neighboring fields (such as Statistics, Computer Science, and Operations Research). In doing so, I hope to make the course valuable to all students, regardless of their current (or future) research interests.

Prerequisites. Basic knowledge of Mathematical Analysis, Linear Algebra, and Matrix Analysis is assumed. Although some prior knowledge of Probability and Statistics helps, it is not strictly necessary. I will try to make the course as self-contained as possible.

Course Material. I will provide you with detailed and self-contained Lecture Notes. You will be responsible only for the material in the Lecture Notes. However, studying good textbooks (see Section 4 for suggestions) improves your understanding of the subject.

Office Hours. Stop by my office whenever I am there or email me to schedule a meeting.

2 Evaluation

To learn the material of this course, you must do more than just read the Lecture Notes and listen to the lectures. Some tips are the following:

- *Active Reading.* Work through each line of the Lecture Notes, be sure you know how to go from one line to the next, and redo and complete the proofs made in class. I expect you to give me feedback on the Lecture Notes (e.g., spot typos and suggest improvements).
- *Active Listening.* Follow each argument we work through in class and ask questions.
- *Working Problems.* Solving the assigned exercises (and more, if you like) is the most valuable part of the course.
- *Working Together.* Working in groups is strongly encouraged, but always try to work through each exercise yourself before talking to others.

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Grading. Problem Sets (25%) and Final Exam (75%).

- *Problem Sets.* I will assign weekly Problem Sets (all relevant instructions via email).
- *Final Exam.* The final exam will be in class, open book, and with no time pressure.

3 Detailed Syllabus

1. Preliminaries of Mathematical Analysis
2. Sample Spaces, σ-Algebras, and Probability Measures
3. Random Variables and Distributions
4. Independence
5. Integration, Mathematical Expectation, and Related Theorems
6. Generating Functions and Their Applications
7. Probabilistic Inequalities
8. Convergence of Random Variables
9. Laws of Large Numbers and Central Limit Theorems
10. Conditional Expectation and Prediction
11. Basics of Information Theory
12. Introduction to Stochastic Processes

4 References

- Billingsley (2012). *Probability and Measure*. Anniversary Edition, Wiley.
- Corbae, Stinchcombe, and Zeman (2009). *An Introduction to Mathematical Analysis for Economic Theory and Econometrics*. Princeton University Press.
- de la Fuente (2000). *Mathematical Methods and Models For Economists*. Cambridge University Press.
- Grimmett and Stirzaker (2020). *Probability and Random Processes*. 4th Edition, Oxford University Press.
- Gut (2005). *Probability: A Graduate Course*. Springer.
- Jacod and Protter (2000). *Probability Essentials*. Springer.
- Ok (2007). *Real Analysis with Economic Applications*. Princeton University Press.
- Ok (2025). *Probability Theory and Measure with Economic Applications*. Available on the author's website.
- Rosenthal (2006). *A First Look at Rigorous Probability Theory*. 2nd Edition, World Scientific.